

Tutorial 1

Recall the following problems:

CLIQUE

Input: Graph G and an integer k .

Output: Does G contain a subgraph H isomorphic to K_k , a complete graph on k vertices?

INDEPENDENT SET (IS)

Input: Graph G and an integer k .

Output: Does G contain an induced subgraph H isomorphic to $\overline{K_k}$, the complement to the complete graph on k vertices?

VERTEX COVER (VC)

Input: Graph $G = (V, E)$ and an integer k .

Output: Is there a subset of vertices $W \subseteq V$ of size $|W| \leq k$ such that every edge has at least one endpoint in W ? Equivalently, is $G \setminus W$ an edgeless graph?

Exercise 1. Show the following polynomial reductions.

- From 3SAT to VC.
- From VC to IS.
- From IS to CLIQUE.

Exercise 2. Prove that INDEPENDENT SET is FPT if and only if CLIQUE is FPT (using the standard parameter – solution size).

What can you say about VERTEX COVER and INDEPENDENT SET?

Exercise 3. Suppose you have a 3SAT solver running in time T whose only output is whether the input formula is satisfiable or not. Can you use it to find a satisfying assignment to the input variables in time $T \cdot \text{poly}(n)$?

* **Exercise 4.** MAXIMUM INDEPENDENT SET has a trivial algorithm running in time $\mathcal{O}^*(2^n)$. Design an algorithm which runs in time $\mathcal{O}^*(c^n)$ for $c < 2$. On the next page, you will find such an algorithm, and your task is to prove its correctness and running time. Will you try to invent your own algorithm?

Algorithm of Exercise 4: The algorithm is recursive:

- Let u be a vertex of minimum degree in G .
- For every $v \in N[u]$, we recurse on $G \setminus N[v]$ while adding v to the solution, and we return the choice which maximizes the solution size.
- If the graph on input has size 0, return $\emptyset$.

Why is this correct?

What is the running time?