Tutorial 2

Exercise 1. In the Point Line Cover problem, we are given a set of n points in the plane and an integer k. The goal is to check if there exists a set of k lines that contain all the input points. Show a kernel for this problem with $\mathcal{O}(k^2)$ points.

Exercise 2. In the Ramsey problem, we are given as input a graph G and an integer k, and the objective is to test whether there exists in G an independent set or a clique of size at least k. Show that Ramsey is FPT.

Exercise 3. In the d-Bounded-Degree Deletion problem, we are given an undirected graph G and a positive integer k, and the task is to find at most k vertices whose removal decreases the maximum vertex degree of the graph to at most d.
Obtain a kernel of size polynomial in k and d for the problem. (Observe that Vertex Cover is the case of $d = 0$.)

Exercise 4. A graph G is a *cluster graph* if every connected component of G is a clique. In the Cluster Editing problem, we are given as input a graph G and an integer k. The objective is to check whether one can edit (meaning add or delete) at most k edges of G to obtain a cluster graph. That is, we look for a set $F \subseteq \binom{V}{2}$ of size at most k such that the graph $(V, (E \setminus F) \cup (F \setminus E))$ is a cluster graph.
Show a kernel for Cluster Editing with $\mathcal{O}(k^2)$ vertices.