# Tutorial 5

***Reminder:*** Vertices $u \neq v$ have the same *type* if $N(u) \setminus \{v\} = N(v) \setminus \{u\}$. $\mathrm{nd}(G)$ is the number of such types in $G$.

**Exercise 1.** Find $\mathrm{vc}(G)$ and $\mathrm{nd}(G)$ for

(a) $G = K_{a,b,c}$, the complete tripartite graph, where $a, b, c \geq 2$,

(b) the disjoint union of $m$ triangles,

(c) the "corona" of $K_r$ ($r \geq 3$): a clique on vertices $c_1, \ldots, c_r$, plus, for each $i$, a new vertex $p_i$ adjacent only to $c_i$.

Which graph families have bounded nd? And which bounded vc?

**Exercise 2.** Find a polynomial kernel for 3-Colouring, parameterized by vc.

**Exercise 3. Hamiltonian Cycle by vertex cover**
In the Hamiltonian Cycle problem, we are given a graph $G$ and want to detect whether $G$ admits a simple cycle through every vertex.
Solve Hamiltonian Cycle in FPT time, parameterized by $\mathrm{vc}(G)$.

**Exercise 4. Two new parameters**
$\mathrm{fvs}(G)$ is the minimum number of vertices $|X|$ such that $G - X$ is a forest.
$\mathrm{dc}(G)$, the *distance to a cluster graph*, is the minimum number of vertices whose deletion leaves a disjoint union of cliques (single vertices allowed).

(a) Prove $\mathrm{fvs}(G) \leq \mathrm{vc}(G)$ and $\mathrm{dc}(G) \leq \mathrm{vc}(G)$.

(b) Show that neither fvs nor dc bounds vc.

*This adds arrows* $\mathrm{vc} \to \mathrm{fvs}$ *and* $\mathrm{vc} \to \mathrm{dc}$ *to the lecture's poset. It is also known that* $\mathrm{tw} \leq \mathrm{fvs} + 1$.