Tutorial 6

Warm up: Feedback Vertex Set

Recall the reduction rules from the lecture:

1. If there is a loop at a vertex v, delete v and decrease k by one.
2. If there is an edge with multiplicity larger than 2, reduce its multiplicity to 2.
3. If there is a vertex v of degree ≤ 1, delete v.
4. If there is a vertex with degree 2, delete it and connect its two neighbors with a new edge.

Lemma A. Let G be a multigraph with minimum degree at least 3. Then, for every *feedback vertex set* X of G, more than half of the edges of G have at least one endpoint in X.

Exercise 1. Derive a randomized FPT algorithm for Feedback Vertex Set.
We know a deterministic $\mathcal{O}^*((3k)^k)$-time algorithm from the lecture. What running time do we achieve here?

Longest Path via Color Coding

Problem: Colorful Path
Input: Graph G, integer k, color $c_v \in [k]$ for every vertex $v \in V_G$.
Output: Does G contain a k-path with all k colors?

Exercise 2. Obtain a deterministic dynamic programming routine for Colorful Path, running in $\mathcal{O}^*(2^k)$ time.

Lemma B. Let U be a set of size n and $X \subseteq U$ its subset. Let

$$\chi : U \to \{1, \ldots, |X|\}$$

be U's uniformly random coloring. Then the probability that the elements of X are colored with pairwise distinct colors is at least $e^{-|X|}$.

Exercise 3. Derive an $\mathcal{O}^*((2e)^k)$-time algorithm that, given a Longest Path instance (G, k), either reports failure or finds a path on k vertices.
On yes-instances, it must succeed with constant probability.